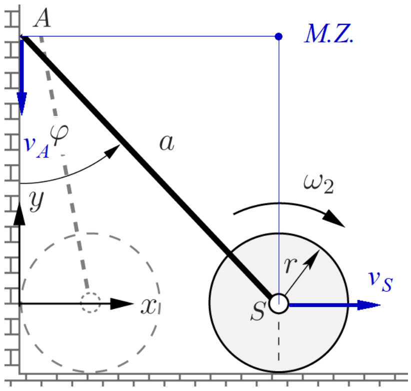
Aufgabe 2.1



a) $x_S(t) = v_S\, t + r \;, \quad \omega_2 = \dfrac{v_S}{r}$

b) $\vec{v}_A = \vec{v}_S + \vec{\omega}_1 \times \vec{r}_{SA}$

$$= \begin{pmatrix} v_S \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\varphi} \end{pmatrix} \times \begin{pmatrix} -a\,\sin(\varphi) \\ a\,\cos(\varphi) \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} v_S - a\,\dot{\varphi}\cos(\varphi) \\ -a\,\dot{\varphi}\sin(\varphi) \\ 0 \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 0 \\ -v_A \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad v_S - a\,\dot{\varphi}\cos(\varphi) = 0 \quad \Rightarrow \quad \dot{\varphi} = \frac{v_S}{a\,\cos(\varphi)}$$

c) $x_S(T) = v_S\, T + r \stackrel{!}{=} a \quad \Rightarrow \quad T = \dfrac{a - r}{v_S}$

$$\varphi = 30^\circ \quad \Rightarrow \quad \cos(30^\circ) = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \dot{\varphi} = \frac{2\,v_S}{\sqrt{3}\,a}$$

$$\varphi = 90^\circ \quad \Rightarrow \quad \cos(90^\circ) = 0 \quad \Rightarrow \quad \dot{\varphi} = \infty$$

Aufgabe 2.2

a) $\ddot{\varphi}(t) = \varepsilon \ , \quad \dot{\varphi}(0) = \frac{v_0}{r} \ , \quad \varphi(0) = 0$

$$\Rightarrow \quad \dot{\varphi}(t) = \varepsilon\, t + \frac{v_0}{r} \ , \quad \varphi(t) = \frac{1}{2}\,\varepsilon\, t^2 + \frac{v_0}{r}\, t$$

$$\varphi(t_1) = \frac{1}{2}\,\varepsilon\, t_1^2 + \frac{v_0}{r}\, t_1 \stackrel{!}{=} \pi + \alpha$$

$$\Rightarrow \quad t_1^2 + \frac{2\, v_0}{\varepsilon\, r}\, t_1 - \frac{2\,(\pi+\alpha)}{\varepsilon} = 0 \quad \Rightarrow \quad t_1 = -\frac{v_0}{\varepsilon\, r} + \sqrt{\left(\frac{v_0}{\varepsilon\, r}\right)^2 + \frac{2\,(\pi+\alpha)}{\varepsilon}}$$

$$\dot{\varphi}(t_1) = \varepsilon\, t_1 + \frac{v_0}{r} = \varepsilon\left(-\frac{v_0}{\varepsilon\, r} + \sqrt{\left(\frac{v_0}{\varepsilon\, r}\right)^2 + \frac{2\,(\pi+\alpha)}{\varepsilon}}\right) + \frac{v_0}{r} \stackrel{!}{=} \dot{\varphi}(t_1) = \frac{v_1}{r}$$

$$\Rightarrow \quad \varepsilon\sqrt{\left(\frac{v_0}{\varepsilon\, r}\right)^2 + \frac{2\,(\pi+\alpha)}{\varepsilon}} = \frac{v_1}{r} \quad \Rightarrow \quad \left(\frac{v_0}{\varepsilon\, r}\right)^2 + \frac{2\,(\pi+\alpha)}{\varepsilon} = \left(\frac{v_1}{\varepsilon\, r}\right)^2$$

$$\Rightarrow \quad v_0 = \sqrt{v_1^2 - 2\,\varepsilon\, r^2\,(\pi+\alpha)} = \sqrt{625\,\mathrm{m}^2\,\mathrm{s}^{-2} - 2\cdot\frac{75}{7\,\pi}\,\mathrm{s}^{-2}\cdot 9\,\mathrm{m}^2\cdot\frac{7\,\pi}{6}} = 20\,\mathrm{m\,s}^{-1}$$

$$a_{N1} = \frac{v_1^2}{r} \approx 208,3\,\mathrm{m\,s}^{-2}$$

b) $x(t) = v_1\,\cos(\alpha)\, t \ , \quad y(t) = -\frac{1}{2}\, g\, t^2 + v_1\,\sin(\alpha)\, t$

$$x(t_2) = v_1\,\cos(\alpha)\, t_2 \stackrel{!}{=} l \quad \Rightarrow \quad t_2 = \frac{l}{v_1\,\cos(\alpha)} \approx 1,85\,\mathrm{s}$$

$$y_2 = y(t_2) = -\frac{1}{2}\, g\, t_2^2 + v_1\,\sin(\alpha)\, t_2 \approx 6,03\,\mathrm{m}$$

$$h_2 = y_2 + r - r\,\cos(\alpha) = y_2 + r\,(1-\cos(\alpha)) \approx 6,43\,\mathrm{m}$$

c) $\dot{x}(t_2) = v_1\,\cos(\alpha) \approx 21,65\,\mathrm{m\,s}^{-1} \ , \quad \dot{y}(t_2) = -g\, t_2 + v_1\,\sin(\alpha) \approx -5,97\,\mathrm{m\,s}^{-1}$

$$v_2 = \sqrt{(\dot{x}(t_2))^2 + (\dot{y}(t_2))^2} \approx 22,45\,\mathrm{m\,s}^{-1} \ , \quad \beta = \tan^{-1}\left(\frac{\dot{x}(t_2)}{|\dot{y}(t_2)|}\right) \approx 74,6°$$